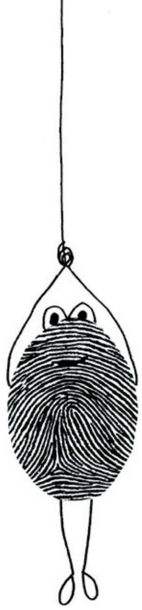


PENDULUM MOTION

People have been using pendulums for thousands of years. Children have been swinging on ropes and vines since ancient times. Some ancient cultures figured out how to rig up baby swings. The Egyptians invented an irrigation device made of a bucket hanging from a beam. Weapons such as slings and trebuchets used swinging objects. During the Renaissance, heavy pendulums were used as sources of power for machines such as saws and pumps. You'd think that over the millennia someone would have noticed how incredibly regular the motion of a pendulum is, and thought of using them for keeping time. But the first written record we have of this discovery is by Galileo, in the year 1602--not that long ago when you consider the entire history of the human race.

The story goes that one day Galileo was sitting in the cathedral of Pisa (the town in Italy that has the famous Leaning Tower). His mind was restless and he turned his gaze toward a chandelier above his head. For some unknown reason, the chandelier was swinging back and forth just a bit. (Perhaps it was a hot day and the doors had been opened to allow air to circulate.) As Galileo watched the motion of the chandelier, he noticed that the swinging motion was very regular. If we had been sitting there, we might have said the swings were like the ticking of a clock. But Galileo would not have thought this. His everyday life had no ticking clocks in it. Clocks did exist, but they were large and expensive. A town might have one clock, placed on the front of a public building so everyone could see it. Many people still used sundials or water clocks. Portable time devices, such as pocket watches, would not exist for another two hundred years. (Even large grandfather clocks were well after Galileo's time.)

A hanging lamp in the cathedral of Pisa, Italy

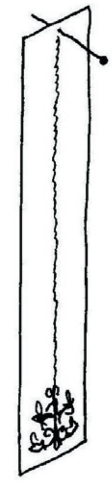
As Galileo watched that chandelier swing back and forth, he began to wonder if the time of each swing would decrease as the chandelier began to slow down. He wished he had a way to time each swing. Then a brilliant thought crossed his mind--he did have a portable timing device he could use! As long as he sat still, his pulse was very regular. He put his finger on his wrist and began to count the time of each swing, using pulse beats as his unit of measure. What he discovered surprised him. The time of the swings stayed the same, even when the swings got very, very small. This was perhaps not the result he had expected. It seemed natural to assume that the short swings would take less time since they were shorter. But no--the time of each swing was identical, no matter how long or short the distance traveled.

You'd think that Galileo would have rushed home and started experimenting with pendulums immediately, realizing that he was on the brink of a major scientific discovery. But he didn't. Perhaps he did a few experiments, but he was a busy college student at this time and did not have enough time to do a thorough investigation of them. It was not until several years later, in 1602, that Galileo was able to do enough research to be able to state with certainty the basic principles of pendulum motion.

You can make the same discoveries that Galileo did. However, you don't have to use the same equipment Galileo used: long ropes, tall ceilings and balls made of lead and cork. You can repeat his experiments using just paper, thread, pins and paper clips.

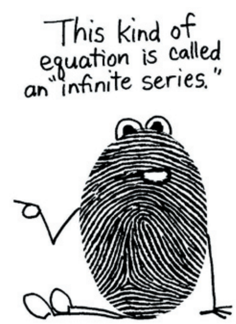
ACTIVITY #1   A swinging chandelier

You will need the chandelier strip (pattern at the back of this packet) printed onto heavy card stock paper, and a pin. (You don't actually need the chandelier strip-- you can use any strip of paper or cardboard.)



1) Print a copy of the pattern page (with the chandelier strip on it) onto heavy card stock paper, then trim it off the chandelier strip. (Or you can use any strip of paper or cardboard.) Save the rest of the paper-- you'll need it for a later activity.
2) Push a pin through the paper right at the top of the chandelier chain (about a centimeter from the top). Wiggle the pin a bit so the hole enlarges just slightly. The strip of paper should be able to swing freely from the pin. Make sure the paper is hanging straight and is not curled at all. Also, make sure you are in a place where the air is very still. If you are near a heater, the paper will be blown about slightly by warm currents of air.
3) Pull the chandelier back and start it swinging. Watch it swing. Hold the pin steady--don't move your hand around. Count the rhythm in your head. (You could even sing a tune to it!) Be very patient and keep watching until the pendulum stops swinging. Watch those tiny swings and keep counting the rhythm.
4) What did you discover about the time of the swings? Did the little swings seem to take as long as the big ones? This is what Galileo discovered. Those tiny swings appear to take just as much time as the bigger ones.

It appears that the distance the pendulum travels back and forth does not affect how long it takes to complete a full swing. You came to this conclusion and so did Galileo. This is entirely true if you don't start the pendulum swinging too high. As long as the swings are relatively small (less than about 10 degrees from the vertical starting point) the timing is so accurate that a pendulum can be used as a time-keeping device inside a clock. However, for larger swings pendulums are not good time-keepers. It turns out that if you release a pendulum from a very high point, those first couple of swings take more time than the smaller ones. Mathematicians can calculate exactly how much difference there is between large swings and small swings This is what their equation looks like:



$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta^4 + \frac{173}{737{,}280}\theta^6 + \ldots\right)$$

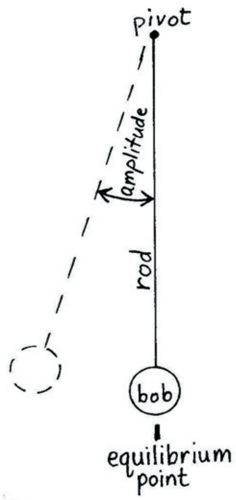
Hmm... let's not delve into that one! Let's just say there is some difference between big swings and little swings, and big swings are not useful for time keeping. If you want to see this difference instead of calculate it, you can do activity #2.

ACTIVITY #2 Watch long swings and short swings

You will need a ruler with a small hole drilled in one end, and a watch or clock. (Some rulers come with pre-drilled holes so you can clip them into binders.) If you don't have a ruler you can drill into, you can use any long, straight, stiff object such as a straw or a pencil. If you can't drill a hole in the end you can just tape a loop of thread to the end. If you can't manage any of these, just try using your chandelier strip and the pin.

1) Put a pin or toothpick through the hole in the end of your rod (or a pencil if the hole is large enough). Make sure the rod swings freely.
2) Pull the rod up to a very high starting point, almost straight up. Let go of the rod and watch it swing. Notice how many large swings it does. Let it keep swinging for a while and observe how many small swings it does.
5) Repeat the experiment.
6) Did the large swings take a bit longer than the small ones? At what point did the pendulum begin ticking regular beats? How long did it keep up the regular beats in comparison to how long it kept doing large swings?

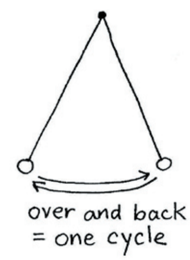
Now it is time to learn the correct names for the parts of a pendulum. The weight at the bottom is called the ***bob***. The place where it hangs is usually referred to as the ***pivot*** and the string is called



the ***rod***. When the pendulum is hanging straight down and not moving at all, it is said to be at its ***equilibrium point***.

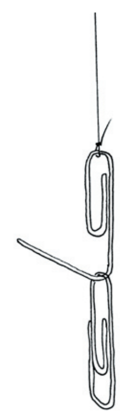
Once the pendulum starts to move, there are names for the aspects of its movement. The size of a swing is called the ***amplitude***. The amplitude is measured in degrees--the same degrees that you use to measure angles in geometry. One complete swing back and forth is called a ***cycle***. The time it takes for a pendulum to complete one cycle (a complete swing) is called its ***period*** (meaning the period of time it takes). And just to confuse you, the number of cycles (swings) per second (or per minute) is called the ***frequency***.

Let's investigate some more ways that a pendulum's swing might be altered. What would happen if we put a very heavy weight on the end? Would it swing more slowly because of the added weight?



ACTIVITY #3 What happens when you change the weight of the bob?

For this experiment we are going to switch to using a pendulum made of thread and paper clips. We need to be able to add weight in a very precise way. You will need a piece of thread, some paper clips, and a watch or a clock with a second hand. If you don't have a stopwatch, but you have a computer, just go to: ***www.online-stopwatch.com***



PART 1

1) Cut a piece of thread about 40 centimeters long (about 16 inches). The exact length is not important.
2) Tie a paper clip onto one end. Open the clip to make a "hook." Put a paper clip on the hook.
3) Use the watch or clock to time how many swings your pendulum makes in 20 seconds. (You might want to write down your result so you don't forget.)
4) Add a few paper clips to the hook and time it again. Write down your result.
5) Add a few more paper clips to the hook and time it again. Write down your result.
6) Really load up the clip this time. Time it again. What is your result?
7) What can you conclude about how weight affects the swinging of a pendulum?

NOTE: You may have some slight differences in the numbers. In all science experiments, there is something called the "margin of error." Since you are human, you are not perfect. Even though it seemed to you that you released the pendulum at the exact moment you heard the word "go," the truth is that you released it a fraction of a second differently each time. You were also off by a fraction of a second each time you called out, "Stop." There may also have been small air currents that were not noticeable to you but nevertheless affected your outcomes each time. Life's little irregularities made their mark on each of your experiments. Therefore, we must expect small errors. Counts that were only one number off (maybe even two) can be considered to be identical results.

PART 2

We will repeat this experiment, but instead of counting how many cycles it completes in 20 seconds, you will count how many cycles it completes from the time it starts swinging until it comes to a complete stop.
1) Load one paper clip onto the hook. Pull the pendulum back just a bit and let it go. Count how many cycles it completes before it comes to a stop.
3) Load ten paper clips (or five if you are short on clips) on the hook, pull it back the same distance you did before and let it swing. Again, count the number of swings it competes before it comes to a stop.
3) Fill the hook with as many clips as you can. Again, count the number of swings it completes before it comes to a stop.
4) What is the effect of a heavier bob?

BONUS EXPERIMENT: Discovering the importance of center of mass

1) Put ten paper clips on the hook and count how many cycles it completes in 20 seconds.

2) Then take those same ten paper clips and attach them end to end, so that they hang down in a long chain below the hook. Count the cycles per 20 seconds. Do the results differ much? Does it matter how that weight is distributed on the pendulum--all in one lump or hanging down in a chain? (See discussion page 10.)

So the weight of the bob has no effect on the period of a pendulum--it only affects how long it will keep swinging. This is perhaps surprising. It might seem that adding weight would slow it down. Many people think that gravity is what slows down a pendulum and makes it stop. They also think that a heavier weight will slow down faster because it will be more affected by gravity. Wrong on both accounts! What slows a pendulum down is what slows down any object: friction. Where can friction be found in a pendulum? There aren't many possibilities, are there? For our chandelier pendulum, we've got the pin rubbing on the inside of the hole, and we have air molecules rubbing against the paper as it swings. That's about it. But that's enough to eventually stop the pendulum.

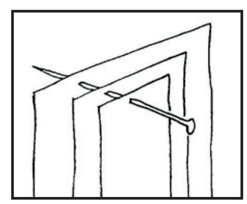
Gravity isn't what stops a pendulum. A friction-free pendulum would go on swinging forever. So what affects the speed at which a pendulum swings?

ACTIVITY #4 Find out what speeds up or slows down a pendulum

You will need your chandelier and the rest of the page it was cut from.

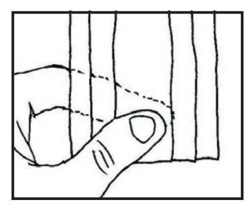
PART 1: Perhaps the width of a pendulum affects how fast it swings? Let's find out.

1) Cut along the remaining line on the sheet from which you cut the chandelier strip. Now you should have three strips of paper that are the same length but different widths. (The thinnest strip will be the chandelier.)



2) Place the medium and large strips under the chandelier strip. Line them up so that the tops and bottoms match up perfectly, and so that they are also centered perfectly. Put the pin through the hole at the top of the chandelier and poke straight through all three strips. Wiggle the pin so that the hole is large enough that all three strips swing freely.

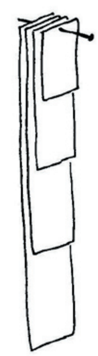
3) Let the strips hang straight down. Spread the tops apart just slightly on the pin so that when they swing they will not be touching each other.



4) Pinch them in the middle (so they don't slip apart) and pull them back. Then let go. They should all start swinging at precisely the same moment.

5) What happens? Do they basically swing at the same rate? (After they swing for a while, there will probably be more difference in their swings. This may be due to the greater effect of air resistance on the wider pendulums, or to some other cause.)

PART 2: Let's try varying the length of the pendulums



1) Use those wide strips to cut three more narrow strips, the exact width of the chandelier strip.

2) Leave the chandelier strip full length. Cut the other three strips so that you have one that is 3/4 the length of the chandelier, one that is 1/2 the length, and one that is 1/4 the length.

3) Line these strips up so that their tops match exactly. Push the pin through that hole in the top of the chandelier chain one last time, then poke it through the other three strips. You should now have four strips of different lengths hanging from your pin.

4) Spread the strips apart so that they can swing freely without touching each other. (If the papers are curled,)run your fingers down them a couple of times and they should straighten out.)

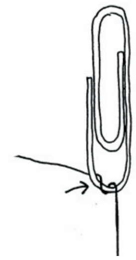
5) Use your finger to pull back all four at once, then let go. What happens? Does the length of a pendulum affect how fast it swings?

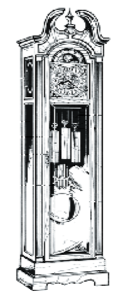
So you've found the secret to making pendulums go faster or slower: adjust the length. You also know how to make a pendulum swing longer: add weight to the bob. You are now ready to make a reliable time-keeping pendulum. Can you make a pendulum that "ticks" seconds, like a clock?

ACTIVITY #5   Make a pendulum that counts seconds

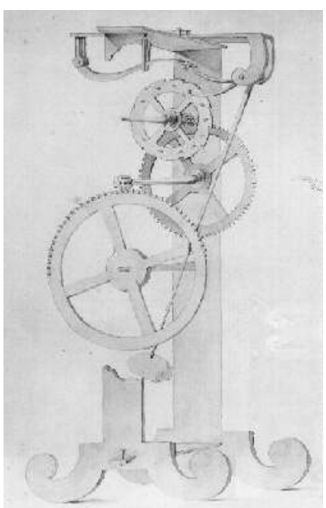
*You will need your pendulum from activity #3, a paper clip, and either an online metronome (try this one: **www.metronomeonline.com**) or a stop watch and someone to say "tick" right on the beat of every second.*

1) Measure the thread. If it is shorter than about 35 centimeters, attach a new piece of thread that is at least 35 centimeters long.
2) Wind the top of the thread around a paper clip, as shown in the picture. This will give you a way to quickly and easily adjust the length of the thread.
3) Hold your pendulum by the paper clip at the top. Put the end of the thread against the paper clip and pinch them together. (You can tape the top of the thread to the edge of a table if you'd rather not hold it. You'll have to keep sticking and unsticking the thread, but this method does work.)
4) Start your pendulum swinging and compare a complete cycle (back and forth) to the seconds ticking on the watch or clock. Is your pendulum faster or slower than one second?
5) Adjust your pendulum height and try again.
6) Keep adjusting until you get a pendulum that ticks off seconds. (You'll find that the longer you let the pendulum run, the more information you will get about its accuracy. It may seem right on for about 10 swings, but after 30 swings it will be half a second off or so.)
7) If you are working in a group and there are other students doing this experiment, compare your results with theirs by having everyone measure the length of their thread. How do the numbers compare? Are they close?

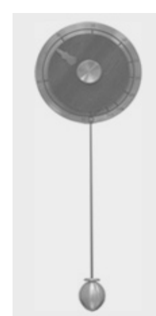




Here's an interesting question-- could you make a working pendulum clock for a doll house? How long would the pendulum have to be if you wanted it to tick off seconds?



One irony in the story of Galileo is that despite his discoveries about pendulums, the idea that pendulums could be used in clocks did not occur to him until the very end of his life, when he had already gone blind and was unable to build one. He described his idea to his son and asked him to make sketches based on his verbal descriptions. His son was able to accomplish this and made very detailed drawings. Galileo died shortly after this and the clock was never actually built. A few years later, a Dutch man named Christian Huygens designed and built his own pendulum clock, and thus received credit as the inventor of the pendulum clock. We don't know if Huygens saw Galileo's sketches before he invented his clock. Huygens was also a brilliant man and could have come up with the idea on his own.



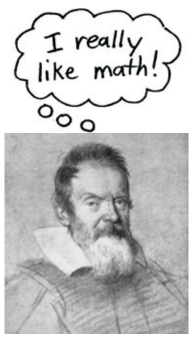
Galileo did find one practical use for pendulums during the time that he was conducting his experiments with them. He invented a machine called a ***pulsilogium*** (*pulse-i-LO-gee-um*). Doctors could use this device to measure the pulse (heart rate) of their patients. This seems almost silly to us, but remember that in Galileo's day portable time-keeping devices did not exist. We take it for granted when we count our heartbeats per minute. Those doctors had no way of measuring a minute. The human memory is not very accurate when it comes to perception of time. Big changes would have been noticeable, but the doctors would not have been able to determine with any accuracy whether a patient's pulse was getting gradually faster or slower over the course of time.

Another practical use for the steady beat of a pendulum is a ***metronome***. This device is used by musicians to help them stay on beat when practicing their music. The pendulum in a metronome is connected to some kind of ticking device that is loud enough for the musician to hear over the sound of their instrument, but not so loud as to drown them out. In the past few decades, metronome manufacturers have switched over to digital, but up until the 1980s metronomes were still mechanical.

ACTIVITY #6   Make a simple metronome

You will need your chandelier strip, some paper clips or binder clips, a pin, and music.

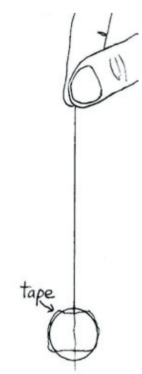
1) Attach some weights to the chandelier, such as a dozen or so paper clips or a few metal binder clips. This will help it rock back and forth more smoothly.
2) Start your music playing.
3) Put your pin into any of the chandelier links, then start it rocking back and forth.
4) Compare the rate that the chandelier is rocking to the beat of the music. If the chandelier is going too fast, shift the position of the pin. (Up or down? Remember what we learned?)
5) Keep shifting the pin until the chandelier is rocking exactly to the beat of the music.



Galileo liked to apply mathematics to his science whenever he could. He thought that perhaps pendulums had some kind of math to them. Perhaps there was a mathematical relationship between the period of a pendulum and the weight of the bob or the length of the rod. He did not know what he was looking for--he just started experimenting and writing down measurements. He did indeed find an interesting mathematical relationship. You can do the same thing Galileo did and find it for yourself. (Although we've "streamlined" the process so you don't have to start from scratch like Galileo did. Your experiments will lead to straight to a conclusion. Galileo's experiments probably took several months to complete. Yours will only take a few minutes.)

ACTIVITY #7   Discover some pendulum math

You will need a long piece of thread (90 centimeters or so), a coin (a penny is fine--that's what I used), a ruler that can measure centimeters, some tape, and a stop watch. (Remember, if you don't have a stopwatch but do have a computer, you can use www.online-stopwatch.com.)



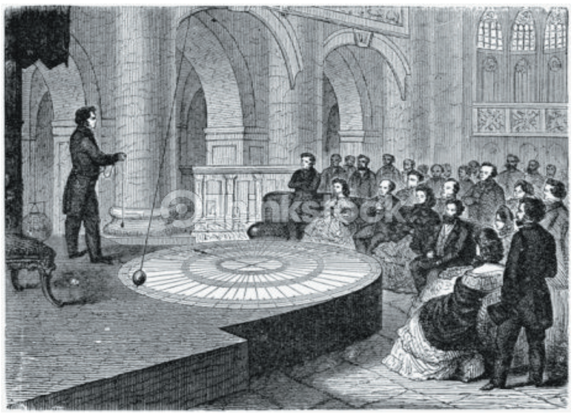
1) Tape the coin to the end of the thread.
2) Pinch the thread at precisely 5 cm above the coin. You need to be as accurate as you can. (You can make a small ink dot on the thread if you find it helpful.)
3) Time this pendulum's cycles for 15 seconds. (One cycle is "out and back.") You may want to do this several times so that you can be sure of your number. This will be the most difficult pendulum to time since it is so short. Record your answer below (where it says 5 cm ___).
4) Now pinch the thread at 20 cm above the coin. This second pendulum will be 4 times as long as the first one. Time it for 15 seconds. Record your answer.
5) Pinch the thread 45 cm above the coin. This pendulum is 9 times as long as the original one. Time it for 15 seconds. (Your answer could be a fraction such as 12.5 (12 and a half) if the pendulum has only completed half its cycle when time is called.)
6) Finally, pinch the thread at 80 cm. This pendulum is 16 times as long as the first one. Time it for 15 seconds.

5 cm _____   20 cm (4 times longer) _____   45 cm (9 times longer) _____   80 cm (16 times longer) _____

7) Can you find a pattern to these numbers? (If your numbers seem really consuCan you predict what the frequency would be if the pendulum was extended to 125 centimeters? ____ To 180 centimeters? ____ Here's a harder one--what about to 50 cm? _____
8) For a hint, look at the bottom of page 8. For the answers, see page 10. (Also, fill in the graph on page 10!)

BONUS EXPERIMENT: More pendulum math
1) Tie a knot anywhere in the thread of your pendulum. Hold this knot and time the pendulum.
2) Fold the thread in half so that the knot touches the middle of the coin. Hold the thread by this halfway point and time this half-length pendulum.

3) Divide frequency of the first pendulum by the frequency of the half-length pendulum.
4) Now use a calculator to find the square root of 2. (You can use Google if you don't have a calculator handy. Just type in "number/number=" and the answer will pop up right underneath.)
5) Compare these two numbers. Close?
6) Try it again with your knot in a different place. Same result?



Another scientist whose name is connected with pendulums is Léon Foucault (*fu-CO*), a French physicist of the 1800s. Foucault mainly studied light, but he is most famous for an experiment he did with a pendulum. He attached a 28-kilogram (60-pound) weight to a 67-meter (220-foot) rope and hung it from the roof of a public building in Paris called the Pantheon. He made marks on the floor showing the direction of the swing of the pendulum. As the day went on, it looked like the pendulum was gradually shifting its direction, rotating away from those marks on the floor. Within the course of 32 hours, it appeared to have made a complete 360-degree circle, finally coming back to its original position.

Foucault's pendulum surprised everyone. They were mystified by its strange behavior. Why did it turn? Foucault explained that the pendulum was not turning--the Earth was. The pendulum continued to swing in the same direction, while the Earth rotated underneath it. This phenomenon would have been easier to understand had the pendulum been on top of the North Pole. You can imagine the Earth spinning on its axis. (In recent years, the Foucault pendulum experiment has been carried out at the South Pole. The pendulum seemed to rotate counterclockwise because the Earth was rotating the opposite direction underneath it.) But why would this phenomenon still occur at other places on the globe? Foucault came up with a mathematical formula that could predict how long a pendulum would take to make a full circle, taking into account how far away it was from the poles as well as some other forces of motion. Pendulums at the equator don't appear to rotate at all.

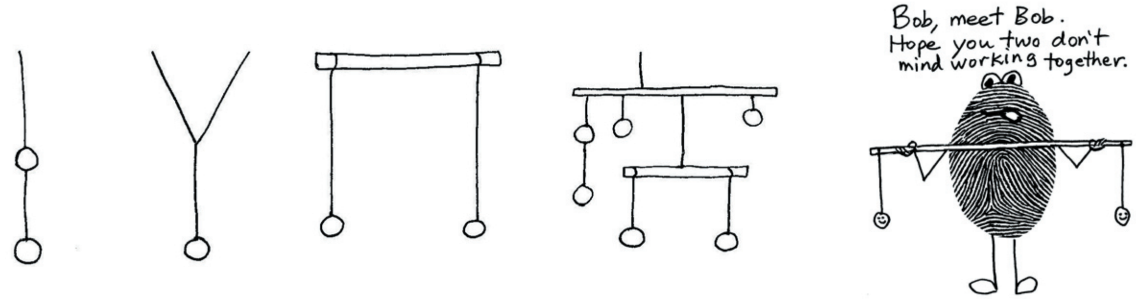
Modern physicist are continuing to research Foucault pendulums. They are fairly sure that these pendulums reflect not only the rotation of the Earth about its axis, but also the revolution of the Earth around the sun. It may be that these pendulums are giving us clues about the motion of the Universe itself!

ACTIVITY #8 Watch some short video clips about Foucault pendulums

Go to www.youtube.com/eejm63 (the Basement Workshop's "safe zone" channel) then click on the "Physical science stuff" playlist. You'll find three videos:
1) A professor at Dartmouth College demonstrates their Foucault pendulum and explains how it works. His explanation is very clear and easy to understand.
2) A "video field trip" to the original Foucault pendulum in Paris. (Actually, the pendulum has been moved and replaced several times, so it's not really THE original.)
3) A time-lapse film showing a whole day of Foucault pendulum motion in just a minute or so.

So far we've experimented with a single pendulum. What would happen if you joined two or more pendulums together? As you might guess, each pendulum would have its own distinct motion, but its motion would also be affected by the motion of every other pendulum it was connected to. What would be the result? Anything interesting, or just a mess?

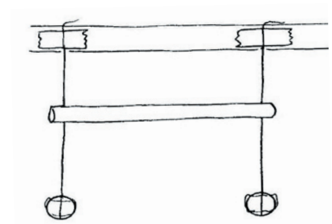
In this next activity, you'll experiment with a ***compound pendulum***. A compound pendulum is made of two or more pendulums attached together in some way. Each pendulum makes its own contribution to a complex final pattern of motion.



Compound pendulums come in many shapes. Mobiles (hanging sculptures) qualify as compound pendulums, although they usually aren't designed for motion.

ACTIVITY #9 Pendulums that take turns

You will need some thread, two coins, tape, a drinking straw and a pair of scissors.



1) Cut two pieces of thread about 40 centimeters (16 inches) long.
2) Tape a coin to the end of each one. (Or perhaps you can recycle your pendulum from activity #7 and use it for one of these pendulums.)
3) Tape the free ends of the threads to the edge of a table. The distance between the threads should be just a little less that the length of the straw.
4) Make snips in both ends of a straw. (If it has a flexible part, trim that off.) Don't cut a "V" just cut a straight slit.

5) Slip the notches of the straw onto the thread to make a bar that goes between the threads, as shown. The straw should at least be several inches (8-10 cm or so) above the coins. If the straw is too close to the coins, it will be too difficult to observe the phenomen we want to observe.
6) Start one coin swinging but don't touch the other one.
Watch for a while and see what happens. It should look like the pendulums are passing their energy back and forth, When one pendulum reaches the height of its energy, the other one will be at rest (just for a split second).
7) You can adjust the height of the straw by sliding the threads through the notches. Try the experiment again with the straw up high or down low.

BONUS EXPERIMENT: "Communicate" with your coins
1) Re-tape the threads so that one thread is substantially longer than the other one. Keep the straw between the threads.
2) Tap or lightly push the straw in a way that makes one coin swing and the other stay still. (You have to discover for yourself how to do this.) Now see if you can do the reverse and get the still coin moving and the moving coin stopped.
3) For a discussion of the results, see the discussion section on the page 10.

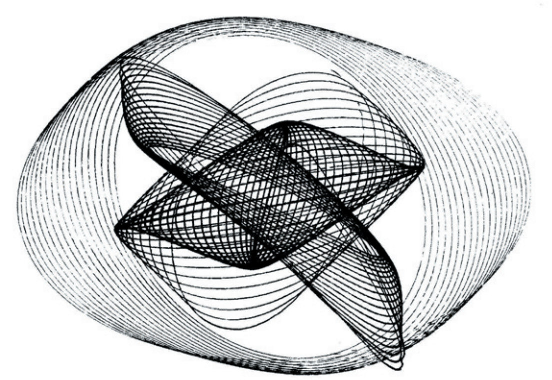
The experiment you just did was essentially a 2-dimensional one. The pendulums went back and forth within a flat plane. You could see motion being transferred back and forth from one pendulum to the other. What would happen if you put pendulums at a right angle to each other? What kind of motion would you get?

In 1844 a Scottish mathematician named Hugh Blackburn began experimenting with compound pendulums and invented something called a ***harmonograph.*** Unfortunately, much of Blackburn's work

HINT for activity #7: This mathematical pattern is called the inverse square law. If your numbers seem confusing, try using the ones I got when I did this activity: 5: 34, 20: 17, 45: 11.3, 80: 8.5

was not well documented (written down) and we don't have any good pictures or models of his pendulums. However, Blackburn's work inspired other mathematicians and physicists to do more investigations, and as a result several types of harmonographs were developed.

In order to be able to understand the complex motions of a compound pendulum, it is necessary to record this information some how so you can see it. One way to do this is to attach pens to the pendulums so that they draw out the shapes onto paper. Computers can also be programmed to simulate pendulums and draw lines electronically on the screen. Some science centers rig up pendulums with funnels that dispense sand onto a table. All of these methods provide a visual record of the motion created by the pendulums. I



This pattern was made by a 3-pendulum rotary harmongraph constructed by the author of this lab.

ACTIVITY #10 Watch a few short videos

Go to YouTube.com/eejm63 and click on the "physical science stuff" playlist. Watch these videos:

1) A video showing a compound pendulum with two pivot points. ("Double Pendulum")
2) The same type of compound pendulum rendered as a computer simulation that traces a bright line to show the path of the bob. ("Double Pendulum Chaos Light Writing")
3) A demonstration of a 3-pendulum rotary harmonograph. ("Three Pendulum Rotary")

The fancy scientific name for pendulum motion is ***oscillation*** *(os-sill-ay-shun)*. Oscillation is any kind of regular back-and-forth motion. Most pendulums oscillate from side to side. But it's possible to have a pendulum oscillate up and down. If you hang a "Slinky" toy the ceiling then get it started going up and down, you've created a "vertical pendulum." Vertical pendulums work in much the same way as horizontal ones. Short ones go up and down quickly (high frequency) and long ones go up and down slowly (low frequency). Light ones come to rest more quickly than heavy ones. However, the springiness of the rod adds another variable that regular pendulums don't have. You can't overstretch a regular pendulum rod the way you can a "Slinky"!

Computers can be used to generate precise mathematical oscillations of all kinds. A machine that shows oscillations is called an **oscilloscope**. If you would like to experiment with computer-generated oscillations and make really neat patterns with them, try the on-line oscilloscope at this address:
http://www.math.com/students/wonders/lissajous/lissajous.html

Discussion of activity #3:

When weight is evenly distributed throughout the pendulum, like in your long chain of paper clips attached end to end, the point that determines the period of the pendulum is the center of mass. In this case, the center of mass for the paper clip chain is right in the middle. If you added a few clips to the bottom of the chain, you would shift the center of mass downward. When the clips were all at the bottom, hanging by a thread that added very little weight, the center of mass as at the bottom. So we see that the concept of center of mass is important in the study of pendulum motion.

Answer for activity #7:

When the length is increased to be 4 times longer, the frequency slows to 1/2 the rate. When the length is increased to be 9 times longer, the frequency slows to 1/3 the rate. When the length is increased to be 16 times longer, the frequency slows to 1/4 the rate.

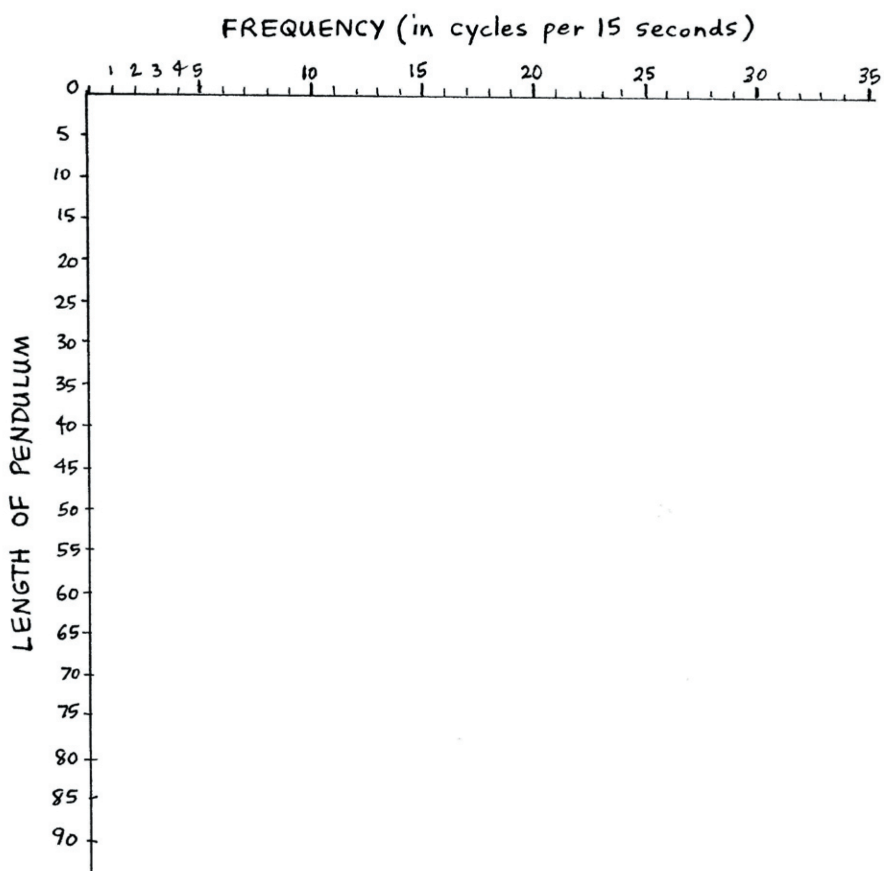
This is called the "inverse square law." When the length is increased N times, the frequency decreases by $1/\sqrt{N}$. (1 divided by the square root of N) 17 is 1/2 of 34, 11.3 is about 1/3 of 34, and 8.5 is 1/4 of 34.

If you extended the length of the pendulum to 125 centimeters that would mean you had made it 25 times as long. The square root of 25 is 5. The frequency would then be 1/5 the rate. 34/5 is 6.8.

If you extended the length of the pendulum to 180 centimeters, you would have made the length 36 times as long. The square root of 36 is 6. So the frequency would be 1/6 of 34. 34/6 is about 5.6

If you made the length 50 centimeters, you would have multiplied the length by 10, which is not a perfect square number. The math is a little messier, but you can do it with a calculator. You'll need to find the square root of 10 (about 3.16). The frequency would be [1/3.16] times slower, or 10.7.

Here is a graph that you can use to plot this information. The points will make a distinct shape. Make a smooth "best fit" curve along the path of these points. After making the curve, you will see that it could be used to estimate the frequency for any length. Put a pencil point on any length number, then move the pencil to the right until it bumps into the curve. Then look up and see what frequency number is right above your pencil point.



Discussion of activity #9:

To get one of the coins swinging, you had to make your taps match its natural period (frequency). It's almost as if each pendulum has a very narrow range of "hearing." It can only "hear" taps that match the frequency at which it would swing if it was in motion. The pendulums "ignore" taps that are not at their special frequency.