CENTER OF MASS

Finding the center of something can often be very important. When you hang a picture, if you get the hanger off-center, the picture hangs crooked. If you are making a top and don't get the handle right

in the center, the top won't spin correctly. The cuts through a pizza had better all go through the center or some people will get smaller slices that others! You can probably think of dozens more examples just by looking at things around you.

You've probably used a ruler to find the center of a line or a square for a math assignment, or perhaps the center of a board or a piece of paper for an art or craft project. Rulers are the most common tool for finding centers. But how accurate are your measurements? In this first activity, you will test your acumen (skill) with a ruler. Can you measure accurately enough to find the exact center of a rectangle? Sounds easy, but is it?



ACTIVITY #1 Use a ruler to find the center of a rectangle, then test your results

You will need a ruler, a pencil, scissors, a piece of heavy card stock paper (or thin cardboard such as the side of a cereal box), and a "balance stand." The balance stand can be made of whatever you have on hand. If you have access to some tools and want to make a permanent one that you can use again in the future, you can

pound a nail up through a piece of wood. Use a fairly large "finishing" nail (with a small head), if possible. You'll need to make sure than the head of the nail on the bottom side is pounded into the wood far enough so that the balance stand doesn't rock back and forth on the head of the nail. The quickest and easiest thing to do is to stick a toothpick into half an apple (put a paper towel under the cut apple, to absorb moisture). This stand won't last for more than a day, of course, but the advantage is that you can recycle it by eating the apple! Or perhaps you could put a sharp pencil into a lump or clay. Use whatever materials work well for you.



1) Use the ruler to draw a square on the piece of card stock or cardboard. The square can be about the size of your palm, or a little larger if you wish. (You can get help making a perfectly square corner by tracing the corner of a piece of paper.) Cut out the rectangle.

2) Use a ruler to find the center of the rectangle and mark it with a dot. (The easiest way to do this is simply to align the ruler so that it connects two opposite corners, then draw a line. Do this to the other pair of opposite corners. The point where these lines cross is the center.)

3) Place your balance stand on the corner of a table. (Putting it on the corner will allow you to look at the underside during the next step.)

4) Place the center dot of your rectangle right on the tip of the balance point (nail or pencil or whatever). Does it balance? If not, move it around until you get it to balance. How far off were you?



Now, on to other shapes. What about circles? Finding the center of a circle can be tricky. It's much easier to start with the center dot, then draw the circle around it using a compass or other circle-drawing device. Compasses often leave a little "pin prick" dot where the point was. You can be sure this little prick is the exact center of the circle, because it was from that dot that the circle was measured off by the compass.

How many objects or machines can you think of that depend in some way on a center dot? (For example, CDs have that center hole that fits onto the spindle.) What would happen to each of these if the hole was off-center? What about finding the center of a triangle? How do you do that? In this next activity you will find the center of both a circle and a triangle.

ACTIVITY #2: Finding the center of circles and triangles

You will need a sharp pencil, a ruler, scissors, your balance stand, another piece of heavy card stock paper (or thin cardboard), and something circular that you can trace around (a CD, a large can, a plate or bowl, etc.)

1) Place the circular object on your cardboard and trace around it with the pencil. (TIP: Put your circular object close to a corner, not in the middle. Putting objects in the middle often leads to excessive waste.)

2) Cut out the circle.

3) Place the ruler across the circle and move it up and down until you find the widest place. Draw a line across the circle at this widest point.

4) Turn the circle a bit then repeat this process. Find the widest place and draw a line.

5) Repeat this process several more times, turning the circle a bit then drawing a line at the widest point. These lines should all intersect at a single point-- the center point.

6) Check your center point with the balance stand. How accurate were you?

7) Draw a triangle on a piece of heavy card stock paper or a piece of thin cardboard. It doesn't have to be a right triangle or an equilateral triangle. Just draw three lines that form a triangle.

8) Cut out your triangle.

9) Use the ruler to find the center of each edge. (If you are using card stock it's thin enough to fold in half.) Mark the centers of each side with a pencil dot.

10) Use the ruler as a straight edge (don't have to measure) to make a connecting line between each center dot and the angle that is directly opposite to it. The place where these three lines meet will be the center of the triangle.

11) Place the triangle on the balance stand and see if it will balance on that center dot.

12) Try another triangle that looks a lot different from your first one.



Scientists have a special term for the balancing point of an object. They call it the *center of mass*. "Mass" refers to... well, it's really complicated. The mass of an object is usually measured using a scale. If someone asked you how much mass you have, you could give the answer in kilograms or

I certainly don't feel like I'm moving!



pounds. However, we can't just say that mass is the same thing as weight. Mass is much trickier than weight. Mass is actually defined as "the measure of an object's resistance to a change in speed." You weren't expecting that, were you?! When you weigh yourself, resistance to speed doesn't exactly come to mind. Fortunately, this technical definition of mass is not essential to understanding the concept of center of mass. Most of the time when we talk about mass, we can use the word "weight" instead. The difference between mass and weight is explained in the lab titled "Inerita and Mass."

So when we find the center of mass, we are basically finding the center of weight-the point around which an object will balance. The center of mass is relatively easy to find for a rectangle or circle or triangle. What about an object with a complicated outline? Can you find the center of mass for any shape?

ACTIVITY #3 Find the center of mass for some odd shapes (first U.S. states, then letters)

You will need copies of the state outlines and the letter outlines (provided at the back of this lesson) printed onto heavy card stock paper, plus a pencil, scissors, a pin, a piece of thread, and a small nut or washer (or some other object with a hole that can be tied onto the end of a piece of thread). If you don't have card stock paper available, you can print the patterns onto regular paper, then trace them onto a sheet of thin cardboard, like a cereal box. 1) Use the patterns to create card stock (or cardboard) state shapes. You can either print the patterns onto heavy card stock paper (the heavier the better) or you can print the patterns onto regular paper, then cut them out and use them to trace out state shapes onto thin cardboard. Either way, the end result should be state shapes that are thick enough that they won't droop when you try to balance them on your balance stand.

2) Let's start with Utah. Just for minute, pretend that the extra bump at the top is not there and put a pencil dot at the center of the main rectangle. Place that dot on the point of the balance stand and see what happens. Now move Utah around until it balances. Put a dot on the correct balance point. Compare the two dots. How does that extra bump affect the balance point? Is the correct point closer to the bump or farther away from it?

3) Now try Nevada. Using what you learned from Utah, take a guess where the center point might be and mark it with a dot. Then balance Utah and mark the correct balance point. How close were you?

4) Next, we'll try Texas, but for this one we'll show you a sneaky trick for finding the center of mass. Cut a piece of thread (about 20 cm) and tie a nut or washer (or other small object with a hole) onto one end. At the other end, tie a small loop. Put the pin through the small loop, then prick the pin into one corner of Texas (any corner is fine), as close to the edge as you can possible get it. Really, really close to the edge! Wiggle the pin a bit so that Texas hangs freely, maybe even swinging just a bit. (If you have trouble with Texas wanting to fall off the pin, hold it against a wall-- but very gently so that it still can swing.) When both Texas and the weight have come to rest, you will need to draw a line where the thread crosses Texas. You can do this easily by making a pencil mark right where the thread crosses the bottom edge, then using a ruler to connect that dot with the pin prick. Now move the pin to a different corner. Any corner will do. Repeat this process and you will get another line across Texas. Do this for at least two more corners so that have four or



more lines. The intersection of these lines will be the center of mass for the shape of Texas. Place this center dot on your balancing stand and see if it balances. If it doesn't, you should only have to make a very small correction. The dot should have been pretty close if you made sure your pin was very near the edge each time.

5) Try this method for some other states.

6) CHALLENGE: Where is the balance point for Florida? Can you get Florida to balance on your stand?

7) Now for the letters! Once again, either print the letters onto heavy card stock paper (the heavier the better!) or print onto regular paper then use them as patterns to trace onto thin cardboard. The end result should be letters that are stiff enough that they don't droop when you go to balance them on your balance stand.

8) Start with the letters F and J. See if you can balance them.

9) Now try the letter L. Does it balance? Mark a dot where the balance point is. Then snip off the portion marked "1" and try again. Now does it balance? Mark the balance point. Snip off the portion marked "2" and try again, marking the balance point. Finally, snip off "3." How does the balance point change? What direction does it go? How are you changing the center of mass by snipping of parts of the letter?

10) Next, try the "V." Does it balance? As with the "L," snip off the number "1" sections then try balancing again. Then snip off the "2" sections. Does it finally balance? Where do you think the center of mass was before you snipped off any sections?

11) Now for the letter "O." Can you balance it? Why not? Where is the center of gravity for the "O"? Now snip off section "1." Can you balance this letter "C"? Snip off the "2" sections. Does it balance now? Where would you estimate the center of mass is? Finally, snip off the "3" sections. Where is the center of mass?

12) Lastly, try the lower case "d." Before you balance it, try to predict where the center of mass will be. Will it balance? After you find, out, predict what will happen if you snip of the "1" section. How will the center of mass change? Then go ahead and try it.

What a "center of mass" adventure we've had! We've discovered that sometimes the center of mass is actually outside an object! Strange, but true. Let's do some more experimenting with objects that have their center of mass outside of themselves. This concept is a key to performing amazing balancing tricks.

In this next activity, we will be balancing a cardboard V not on your balance stand, but on a tightrope made of thread. You will discover for yourself another basic principle of balancing.





ACTIVITY # 4 Balancing the letter V

You will need a pair of scissors, a piece of thread or thin string (20 cm is plenty), and a cardboard letter V with very long "arms."

1) Cut a cardboard V that has very long "arms." We will be trimming these arms bit by bit, so make sure they are pretty long to start with. The exact length or width is not important. Just cut something similar to what you see in this drawing.

2) Have an assistant hold the ends of the string and pull it tight. Then put the V upside down on the string, so that it balances. Rock it a bit and see how well it maintains its balance. Remember what you discovered about the letter V in the previous activity. Where would the center of mass be for this V?

3) Now trim a bit off each "arm" of the V. Make sure you trim the same amount off each side. Try balancing it again. Does it still balance?



4) Trim the V again, the same amount off both sides. Balance it again. Rock it back and forth a bit. Is it still stable?

5) Keep trimming off a bit from both sides and checking how it balances each time. What happens when the arms get very short? Keep trimming until you reach a point where it does not balance anymore.

6) Take this unstable V and balance it on your balance stand. Does it balance? Is the center of mass inside or outside this V?

7) Where do you think the center of mass was for your original long-armed V? What happens as the center of mass gets closer and closer to the point of the V?

So now we've discovered that the way to achieve vertical balance is to keep the center of mass below the balance point. (The string was the balance point in our last activity). Science museums in big cities sometimes have an exhibit where you can ride a bicycle on a tightrope, often very high up in the air. This can be done safely by making sure that there is a huge amount of weight well below the wire-- so much weight that the bicycle can't possibly become unbalanced. This picture shows a famous "daredevil" act where a man got his wife to act as the center of mass below the rope. It's the same principle as the bicycles at science museums.

If you watch gymnasts do routines on the balance beam, you will notice that if they think they are starting to fall, they will quickly lower their bodies. They need to get their center of mass closer to the balance point in order to gain stability.

Now it's time for some tricks. Here are some balancing acts that use the principle of keeping the center of mass below the balance point.



ACTIVITY #5 The classic fork balancing act

You will need two forks, a toothpick and a glass. Since these objects are often on tables at restaurants, this might be a good trick to do while you are waiting for your meal.

- 1) Put the two forks together so that their tines interlock.
- 2) Wedge the toothpick into a crack where the forks intersect.

3) Rest the toothpick on the edge of a glass or other object. If you need a video demo, there is one posted on the "Physical science stuff" playlist on the Basement Workshop channel: www.YouTube.com/eejm63 (The part about burning the toothpick is optional-- I didn't burn mine.)



ACTIVITY #6 Balance a ruler with a hammer

You will need a hammer, a one-foot ruler (30 cm), and some string or a strong rubber band.

This balancing act looks like it should not be possible. The hammer hangs precariously underneath the ruler, looking like it should make the ruler fall off the table. But there it hangs, perfectly balanced!

1) Take the rubber band or string and make a loose loop around the hammer and ruler, as shown in the picture.

2) Make sure the end of the hammer is touching the ruler, and then position the ruler at the edge of a table, as shown. (You might have to reposition the string/rubber band a few times to get it just right.)



3) Why does this trick work? Analyze where the center of mass might be. Where is the balance point? What is the heaviest part of a hammer?



These tricks are interesting and fun, but they're just. tricks -- not very useful for any real purpose. Is this "center of mass" stuff actually important for REAL science or engineering?



These toys use the principle of center of mass.

Yes, of course! Center of mass is important in many branches of science and engineering. Engineers who design vehicles are very concerned about center of mass. Cars that have their center of mass closer to the ground are less likely to flip over (a special concern for racing cars that go around turns at very high speeds). If an airplane has its center of mass too far forward, it will tend to be less

maneuverable and be difficult to control during take-off and landing. A plane with a center of mass too far back will be very maneuverable but will be less stable during flight. A helicopter must be designed so that the center of mass can shift backward when the pilot wants to go forward. (This is why a helicopter looks like it is tilting nose down when it is flying forward-- the center of mass moves behind the rotor.)



In sports, high-jumpers learn to bend in such a way that their bodies go over the bar even though their center of mass does not. Long jumpers shift their center of mass as efficiently as possible in order to maximize the power of their jump. Athletes shift their center of mass all the time as they run and jump, though they don't think about what they are doing in scientific terms. Scientists called kinesiologists (kin-EE-see-OL-o-gists) study body movement and develop equipment or therapies based on the science and math of body movement.

In astronomy, center of mass is very important. When a moon orbits a planet, or a planet orbits a star, they are actually both moving around a central point called the "barycenter." This point can be inside one of the bodies, or at a point in space. The Earth and the moon orbit each other at a point that is about 1,710 kilometers (1062 miles) below the surface of the Earth. It is this barycenter point that orbits around the Sun.



You don't have to be an athlete to experiment with shifting your center of mass. Here are three "body activities" where center of mass plays an important role.

ACTIVITY # 7 "The Impossible Hop"

You won't need anything special for this experiment, but some people like to place a dollar bill on the ground and make a bet with someone that they can't jump over it. (You'll win the bet and keep the dollar bill most of the time.)

1) Bend over and hold on to the tips of your shoes (or your toes if you are wearing just socks).

2) Try to jump forward without letting go of your shoes/toes. (In the dollar bill version, you place the dollar bill in front of their feet and tell them to jump over the dollar without letting go of their shoes/toes.)

3) Most people find this trick impossible because when you jump you shift your center of mass forward. In order to compensate for this sudden shift forward, you instinctive counterbalance using your arms. If you can't counterbalance with your arms because you are holding on to your toes, you will fall over.

ACTIVITY #8 Immobilize someone with your pinky finger

You will need a chair and a volunteer for this activity.

- 1) Tell your volunteer to sit in the chair, with his/her back against the back of the chair and hands in lap.
- 2) Place your pinky finger on his/her forehead. (You can use any finger-- it does not have to be the pinky.)
- 3) Tell your volunteer to stand up. Use your finger to keep his/her head from tilting forward.
- 4) Your volunteer will probably not be able to stand up.

When you are standing, your center of mass (somewhere in your abdomen) is directly over your feet. When you are seated, your center of mass is above the seat of the chair, not over your feet. In order to stand up you need to move your center of mass from over the chair to over your feet. To accomplish this, you need to lean forward. Strangely enough, the amount of force needed to keep someone from leaning forward is not all that much. You can exert this force with one finger.

ACTIVITY # 9 The "girls always win" chair lifting challenge

You will need two volunteers (one male and one female), a chair, and a wall against which you can set the chair. The older the volunteers are, the better. The difference between the center of mass for boys and girls (under the age of 12 or so) might not be enough to make this trick work.

1) Place the chair against the wall.

2) Have one volunteer bend over the chair so that his/her head touches the wall and his/her upper body is parallel to the floor, as shown in the diagram. Tell the volunteer to try to lift the chair and then stand up.

Men and women have their center of mass in different places. Men tend to have broad shoulders and narrow waists, giving them a higher center of mass. In this case, a higher center of mass is a disadvantage because adding the weight of the chair makes the center of mass even higher, creating an impossible situation where there just isn't enough counterbalancing weight in the lower half of the body. Girls and women have their center of mass closer to their hips, which in this case is an advantage because even with the added weight of the chair, the overall center of mass is close enough to being over the feet that the body is able to right itself.

(NOTE: In our classroom, we found it hard to get the "correct" results. Our boys could lift the chair. You might want to try a relatively small chair with a book or two sitting on it.)



You have probably seen pictures of the Leaning Tower of Pisa. It didn't start out as a leaning tower, of course. It was built to be the bell tower of the cathedral in Pisa, Italy. Over the course of hundreds of years, one side has gradually sunk into the ground. What the builders didn't know was that the site they chose to build on had different types of rocks and soil underneath it. So basically, one part of the building was on harder rock and one part on softer rock. The softer rock started to compress and sink under the weight of the building, causing the tower to lean.

In the 1990s, it was determined that the Leaning Tower was leaning so much that is was no longer safe for people to be inside of it. Architects measured the rate at which the lean increased each year and calculated that eventually the tower would fall over. It would still be quite a few years until that happened, but they decided to go ahead and fix it now instead of waiting until the tower got any more dangerous. They figured out a way to adjust the foundation underneath the tower so that the higher side was brought down a bit. In 2004, the tower was declared to be safe . Tourists can now climb the stairs to the top and look out over the city.





How can you predict when an object is leaning enough that it will fall over? It's a matter of calculating where the center of mass is, then determining whether that center of mass is located over the base of the object. In this series of drawings, the dotted line shows where the center of mass is located over the base. In the first drawing, the center of mass is directly over the center of the base. In the third drawing, the building has leaned so much that the center of mass is no longer over

any part of the base, but over the grass. When the center of mass of an object shifts so that it is no longer over any part of the base, the object falls over.

In these two drawings of the Leaning Tower, we've proposed a somewhat ridiculous scenario of what would happen if a whole bunch of tourists climbed to the top of the lower side of the tower. This isn't meant to be something that would really happen, but a way to introduce the concept of center of mass in an object that has areas of unequal density. So far, we've assumed that the weight of the tower is distributed equally, with no part of it being much lighter or heavier than any other part. However, if one section of the



tower was made of Styrofoam, and other parts were made of lead, think of how that would affect the calculation of its center of mass. When calculating center of mass, you have to take into account any areas that heavier (denser) than others. In our little cartoon here, we have a whole bunch of tourists on the low side of the tower. The weight of the tourists has greatly increased the weight of that side of the tower, causing a shift of the center of mass. With the additional weight of the tourists, the center of mass has moved closer to the edge of the base, and, therefore, closer to possible tragedy.



Which one of these guys should not be standing? (Assume that their center of mass is right in the middle of their body and that their base is the area under their feet.)

ACTIVITY #10 The leaning beverage can trick

You will need an empty can of the type shown in this picture. The can must have a bottom that looks like this one, with an angular section between the side and the flat bottom. You'll also need some water that you can gradually pour into the can. (You could also do this trick by gradually drinking the soda until the can will balance.)

1) Start by trying to balance the empty can. (If you happen to have a full can, try to balance that as well. Prove to yourself that neither will balance.)

2) Pour water into the can until it is about 1/3 of the way full. Try to balance it again. If it does not balance, add or subtract a little bit of water until the can balances.





If the can has too little water, the weight of the can itself will be the most significant factor in the center of mass, causing the center of mass to be too far away from the base. In this case, the "base" is that very small edge that the can is balancing on.



If the can has too much water, the center of mass will again be to one side of the balancing point, causing it to topple over.



If the can has just the right amount of water in it, the center of mass will be directly over the base.

ACTIVITY #11 Watch a video of Julius Sumner Miller-- a well-known lecturer of the late 1900s

Julius Sumner Miller is a geeky little professor with more enthusiasm for physics than you can imagine. He was one of the first educators to use "hands on" science with his students. He did several series of physics demonstrations for television during the 1960s to 1980s. He is famous for his opening line, "I'm Julius Sumner Miller, and physics is my business." Early in his career, Miller studied under Albert Einstein at Princeton, and remained a friend until Einstein's death.

You can watch his lecture on "Center of Gravity" (same thing as center of mass) by either searching YouTube for "Julius Sumner Miller Center of Gravity part 1" or by going to the Basement Workshop channel: www.youtube.com/eejm63 and finding this video in the "Physical Science Stuff" playlist. Yeah, he's a little crusty and a bit rigorous, but he's also pretty entertaining and very, very good at what he does!

ACTIVITY #12 One last trick-- balancing a coin on a dollar bill

Did you know you can balance a coin on a paper bill? The trick is to fold the dollar a bit, rest the coin on the fold corner, then slowly open the bill. If your hands are steady and you pull the ends of the bill slowly, the coin will balance right on the edge! If you want to see a video demonstration of how to do the trick, you can find one in the "Physical Science Stuff" playlist on the Basement Workshop YouTube channel.



COPY THIS PAGE ONTO HEAVY CARD STOCK PAPER

NOTE: These states are not drawn to scale. Vermont and New Jersey are much smaller than Texas and Oklahoma! In this activity, we are interested in just their shapes, not in their relative sizes. We could have used other odd shapes, but state shapes are fun to work with.



Z



